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On one solution of the problem of transverse oscillations of a beam with movable boundaries

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Abstract: The problem of oscillations of bodies with movable boundaries, formulated as a differential equation with boundary and initial conditions, is a non-classical generalization of a hyperbolic problem. To facilitate the construction of a solution to this problem and to justify the choice of the solution type, equivalent integro-differential equations with symmetric and non-stationary kernels and non-stationary integration limits are constructed. The advantages of the integro-differential equation method are revealed when moving to more complex dynamic systems carrying concentrated masses oscillating under the action of moving loads. The method is extended to a wider class of model boundary value problems that take into account bending rigidity, resistance of the external environment, and the rigidity of the base of the vibrating object. The solution is given in dimensionless variables with an accuracy of up to the values of the second order of smallness of relatively small parameters characterizing the velocity of the boundary. An approximate solution is found to the problem of transverse vibrations of a viscoelastic beam with bending rigidity, taking into account the action of damping forces.

Keywords: vibrations of systems with moving boundaries, laws of boundary motion, integro-differential equations, amplitude of beam vibrations.

In the field of elastic system dynamics, of particular practical interest are problems related to vibrations of structures whose geometric parameters change over time. Typical examples of such systems are rope mechanisms of lifting devices, flexible elements of transmission mechanisms, drilling rigs, etc. Numerous studies in the field of hoist rope dynamics have revealed the need to develop new approaches to analyzing the behavior of one-dimensional objects with variable geometric characteristics. From a mathematical point of view, such problems require solving hyperbolic equations in domains with changing boundaries. Significant difficulties that arise when describing such systems determine the preferential use of approximate methods of analysis. Among the analytical approaches, the most effective are methods based on special transformations of variables, as well as methods using the principle of superposition of counter wave processes. The approach, which involves the use of complex-valued substitutions of variables, which make it possible to reduce the original problem to an analysis of the Laplace equation, deserves special attention. However, the capabilities of exact analytical methods are significantly limited. Among the approximate methods, the Kantorovich-Galerkin method [1] deserves special attention, as well as the approach developed in this study, based on constructing solutions of integro-differential equations [2, 3]. The proposed method of transforming the initial differential formulation of the problem into an integro-differential form with variable parameters allows to significantly simplify the process of finding a solution. A feature of the developed approach is its efficiency in the analysis of complex dynamic systems containing concentrated masses and subject to the action of moving loads. The paper shows that in the simplest cases, the methods of integral equations do not have significant advantages over classical differential approaches in the study of systems with an infinite number of degrees of freedom. However, when moving to more complex mechanical objects, such as variable-length beams, the advantages of the integro-differential approach become obvious. The proposed method was successfully applied to a wider class of boundary value problems that take into account the rigidity of the structure during bending, the resistance of the external environment and the elastic properties of the foundation. Particular attention is paid to the analysis of the most common case in practice of the impact of external disturbances on the boundaries of the system. It is shown that for fixed boundary conditions, the constructed integro-differential equations are naturally transformed into classical Fredholm equations of the second kind.

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