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Mathematical modeling of nonlinear vibrations of a rope with a moving boundary

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Abstract: The problems of longitudinal – transverse vibrations of objects with moving boundaries were solved mainly in a linear setting, the energy exchange through the moving boundary and the interaction between longitudinal and transverse vibrations were not taken into account until now. The paper presents a new nonlinear mathematical model of transverse vibrations of an unbounded rope with a moving boundary, in which geometric nonlinearity is taken into account. The obtained mathematical model allows one to describe high-intensity vibrations of a rope with a moving boundary. The solution was made in the MATLAB environment of dimensionless variables, which allows one to use the obtained results to calculate oscillations of a wide range of technical objects

Keywords: nonlinear mathematical model, vibrations of a rope, moving boundaries.

The problems of longitudinal – transverse vibrations of objects with moving boundaries were solved mainly in a linear setting, the energy exchange through the moving boundary and the interaction between longitudinal and transverse vibrations were not taken into account until now [1–5, 7–14]. The action of the forces of the external environment's resistance was considered in rare cases [6]. Real technical objects are much more complicated, for example, with an increase in the intensity of oscillations, the geometric nonlinearities of the object have a great influence on the oscillatory process.

It became possible to more accurately describe complex mathematical models of longitudinal-transverse oscillations of objects with moving boundaries with a large number of factors influencing the oscillatory process in connection with the intensive development of numerical methods.

The paper presents a new nonlinear mathematical model of longitudinal-transverse vibrations of a rope with a moving boundary, which takes into account geometric nonlinearity, energy exchange across the boundary. The boundary conditions are obtained in the case of interaction between the parts of the object to the left and to the right of the moving boundary.

Thus, a new nonlinear mathematical model of transverse vibrations of an unrestricted rope with a moving boundary has been developed. It is solved numerically in the MATLAB environment. The boundary conditions are obtained in the case of occurrence between the parts of the object to the left and to the right of the boundary. The obtained model is linearized, while the principle of homogeneity is observed: in the particular case of small fluctuations, the obtained model coincides with the classical one, which indicates the correctness of the results obtained. The obtained mathematical models make it possible to describe high-power oscillations with moving boundaries.

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