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Modeling the behavior of a bar under creep conditions and under finite deformations*

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The paper considers a metal rod of circular cross-section, the length of which is many times greater than its cross-section. A constant tensile force is applied to the rod, which sets the stress p at the initial moment of time. The rod is in creep conditions. In this one-dimensional case, the equilibrium equation takes the form

$$\frac{\partial \sigma}{\partial x} = 0,$$

therefore, the stress will be an arbitrary function that depends only on time: $\sigma = \sigma(t)$. Taking into account that at the initial moment $\sigma(0) = p$, we take this function in the simplest linear form $\sigma(t) = p + kt$, where k is some constant. In areas where irreversible deformations are absent, stresses can be calculated as

$$\sigma_{ij} = \frac{\rho}{\rho_0}\frac{\partial W}{\partial d_{ij}}(\delta_{ij} - 2d_{ij}),$$

where W is the elastic potential, ρ_0 is the density of the material in the undeformed state ($d_{ij} = 0$, temperature $T = const$), δ_{ij} are the Kronecker symbols. In an isothermal process deformation $W = W(d_{ij})$ and for an isotropic medium this function depends only on the invariants of the tensor $d_{ij} : W = W(J_1, J_2, J_3)$. It was shown in [1] that if this function is expanded in a Taylor series with respect to the free state, limiting ourselves to terms up to the third order in the components d_{ij}

$$W = \frac{\lambda}{2}J_1^2 + \mu J_2 + lJ_1J_2 + mJ_1^3 + nJ_3 + ...$$

$$J_1 = d_{ii}, J_2 = d_{ik}d_{ki}, J_3 = d_{ik}d_{kj}d_{ji},$$

then we obtain the following dependence for calculating the stresses in the regions of reversible deformation

$$\begin{gathered}\sigma_{ij} = (\lambda d_{kk} + l d_{st}d_{ts} + (3m - \lambda)d_{kk}^2 - (l + \lambda)d_{kk}d_{st}d_{ts} + (\frac{1}{2}\lambda- \\ -3m)d_{kk}^3)\delta_{ij} + 2(\mu + (l - \lambda - \mu)d_{kk} - (l + 2\mu)d_{st}d_{ts} + (\mu - 3m- \\ -l + \lambda)d_{kk}^2)d_{ij} + (3n - 4\mu - (4l + 3n - 4\mu)d_{kk})d_{it}d_{tj} - 6nd_{it}d_{tp}d_{pj}.\end{gathered} \tag{1}$$

In (1) λ, μ, l, m, n are elastic constants of the medium, and the first two are the Lame parameters: $\lambda = \dfrac{\nu E}{(1 - 2\nu)(1 + \nu)}, \mu = \dfrac{E}{2(1 + \nu)}, \nu-$ Poisson's ratio.

If in the expression for stresses in reversible deformation regions (1) we take into account only the linear dependence on the component of the total strain tensor d: $\sigma = (\lambda + 2\mu)d = p + kt$, then we get

$$d = \frac{p + kt}{\lambda + 2\mu} = \frac{\partial u}{\partial x} - \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2. \tag{2}$$

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Let's introduce the replacement

$$\frac{\partial u}{\partial x} = z, \tag{3}$$

then equality (2) takes the form

$$z^2 - 2z + \frac{2(p+kt)}{\lambda+\mu} = 0.$$

This quadratic equation has roots

$$z_{1,2} = 1 \pm \sqrt{1 - \frac{2(p+kt)}{\lambda+\mu}},$$

from which it is reasonable to take positive. In what follows, we will need the ε - component of the total strain rates, equal to

$$\varepsilon = \frac{dz}{dt} = -\frac{k}{\sqrt{(\lambda+2\mu)(\lambda+2\mu-2p-2kt)}}. \tag{4}$$

If we take into account the quadratic dependence of the total strain tensor component d in the expression for stresses in the regions of reversible deformation (1), then we obtain the relation

$$\sigma = (\lambda+2\mu)d + 3(l+m+n-\lambda-2\mu)d^2 = p+kt$$

or the equation $a_1d^2+a_2d-a_3=0$, where $a_1 = 3(l+m+n-\lambda-2\mu)$, $\quad a_2 = \lambda+2\mu$, $\quad a_3 = p+kt$.

Introducing the notation $b = \dfrac{a_2}{2a_1}$, $c = \dfrac{a_3}{a_1}$, we rewrite this equation in the form $d^2+2bd-c = 0$, the positive root of which will satisfy the relation

$$d = b + \sqrt{b^2+c} = \frac{\partial u}{\partial x} - \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2,$$

which, taking into account the replacement (3), can be written in the form

$$z^2 - 2z + 2(b+\sqrt{b^2+c}) = 0.$$

We are satisfied with the root of this equation $z = 1 + \sqrt{1-2(b+\sqrt{b^2+c})}$, then the component of the total strain rates will take the form

$$\varepsilon = \frac{dz}{dt} = -\frac{k}{2a_1\sqrt{1-2(b+\sqrt{b^2+c})}\sqrt{b^2+c}}. \tag{5}$$

In our case, the equation of transfer of elastic deformations [1] takes the form

$$\frac{de}{dt} = (\varepsilon - \gamma^c)(1-e).$$

The creep of the rod is modeled by kinetic equations; therefore, the deformation process will be described by the following system of equations:

$$\begin{cases} \dfrac{de}{dt} = (\varepsilon - \gamma^c)(1-e), \\[2mm] \dfrac{dp^c}{dt} = (1-2p^c)\gamma^c, \\[2mm] \dfrac{d\omega}{dt} = \dfrac{B}{A}\gamma^c. \end{cases} \tag{6}$$

Here

$$\gamma^c = \frac{A\sigma^n}{\omega^n(1-\omega)^n},$$

the p^c – component of the irreversible creep strain tensor; the γ^c – component of the creep strain rate; ω – parameter of the rod material damage; A, B, n – parameters of creep, ε – the component of the total strain rates can be calculated by the formula (4) or (5). We believe that the stresses in the bar, as in the classical theory, are completely determined by reversible deformations. At the initial moment of time $t = 0$, the rod is in an undeformed state, and the applied stress in a short time τ increases from 0 to p, and the initial conditions will be homogeneous. However, the τ interval is much shorter than the creep time t_* of the material, so the initial conditions can be taken as:

$$e(0) = \frac{p}{E}, \quad p^c(0) = 0, \quad \omega(0) = 0, \tag{7}$$

where E is the modulus of elasticity of the rod material. The system of equations (6) should be solved under the initial conditions (7) until the moment of time $t = t_*$, at which the damage parameter takes the value $\omega(t_*) = 1$, which determines the failure of the rod. The results were compared with the data obtained in [2] when studying the creep of rods with a diameter of 0.042 m made of steel grade St. 45.

References

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