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Dynamical systems on $\mathbb{T}^2$ modeling Josephson junction isomonodromic deformations and Painlevé 3 equations*

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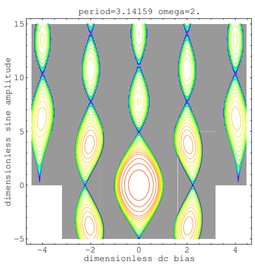
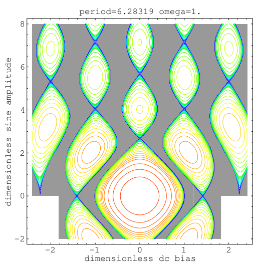
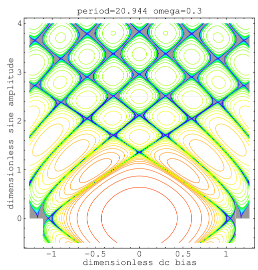
The tunneling effect predicted by B.Josephson (Nobel Prize, 1973) concerns the Josephson junction: two superconductors separated by a narrow dielectric. It states existence of a supercurrent through it and equations governing it. The *overdamped Josephson junction* is modeled by a family of differential equations on 2-torus depending on 3 parameters: B (abscissa), A (ordinate), ω (frequency). We study its *rotation number* $\rho(B, A; \omega)$ as a function of (B, A) with fixed ω. The *phase-lock areas* are the level sets $L_r := \{\rho = r\}$ with non-empty interiors; they exist for $r \in \mathbb{Z}$ [1]. They are analogues of the famous Arnold tongues. Each L_r is an infinite chain of domains going vertically to infinity and separated by points called *constrictions* (expect for those with $A = 0$). See the figures below for $\omega = 2,\ 1,\ 0.3$.







We show [2] that: 1) *all constrictions in L_r lie in the vertical line $\{B = \omega r\}$*; 2) *each constriction* is *positive*: some its punctured neighborhood in the vertical line lies in $\mathrm{Int}(L_r)$. These results confirm experiences of physicists (pictures from physical books of 1970-th) and two mathematical conjectures.

The proof uses an equivalent description of model by linear systems of differential equations on $\overline{\mathbb{C}}$ [1], their isomonodromic deformations described by Painlevé 3 equations and methods of theory of slow-fast systems.

References

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2. Bibilo Yu., Glutsyuk A. On families of constrictions in model of overdamped Josephson junction and Painlevé 3 equation. Preprint. 2021. URL: https://arxiv.org/abs/2011.07839

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