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On synchronization of oscillations in pendulum-type equations under quasiperiodic perturbations *

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We consider a pendulum-type equation close to an integrable one

$$\ddot{x} + \sin x = \varepsilon[(-1 + p_1 \cos 3x)\dot{x} + p_2 \cos \omega_1 t \sin \omega_2 t], \tag{1}$$

where p_1, $p_2 > 0$ are parameters, ε is a small positive parameter, ω_1 and ω_2 are incommensurable frequencies (this implies that the perturbation is a quasiperiodic function in t).

The equation is equivalent to the system

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -\sin x + \varepsilon[(-1 + p_1 \cos 3x)y + p_2 \cos \theta_1 \sin \theta_2], \\ \dot{\theta}_1 = \omega_1, \\ \dot{\theta}_2 = \omega_2. \end{cases} \tag{2}$$

For a perturbed autonomous case ($\varepsilon \neq 0$, $p_2 = 0$), the problem of limit cycles is solved. The solution of this problem is carried out by analyzing the Poincaré–Pontryagin generating functions, the simple real zeros of which correspond to the rough limit cycles of a perturbed autonomous system [1].

For nonautonomous case ($p_2 \neq 0$), the question of the structure of the resonance zones, to which the solution of the problem of synchronization of oscillations leads, is studied. Resonance levels in the unperturbed system ($\varepsilon = 0$) are found. In the neighborhood of an individual resonance level (called an individual resonance zone), we obtain an averaged system that describes the topology of an individual resonance zone up to terms of order ε. A simple stable (unstable) equilibrium state of the averaged system corresponds to a stable (unstable) quasiperiodic resonance solution and two-dimensional invariant tori with quasiperiodic motion in the original four-dimensional system.

We distinguish splittable and nonsplittable resonance levels. Splittable resonance levels are divided into passable, partially passable and impassable ones.

Next, introducing the "detuning", which determines the deviation of the resonance level from the level generating the limit cycle in the autonomous system, bifurcations of the transition from impassable resonance to partially passable one (the phenomenon of passing a three-dimensional torus through the resonance zone) are studied.

The results obtained are illustrated by numerical computations.

This work follows the works [2, 3].

References

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