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Variational statement of the problem of vibrations of a beam with a moving boundary

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Abstract: The statement of the problem of vibrations of a beam with a moving spring–loaded support carrying an attached mass is obtained. When the support is not absolutely rigid, energy exchange occurs through the moving boundary. In this regard, there is a difficulty in writing the boundary conditions. To formulate the problem, we used the variational principle of Hamilton. In this case, the viscoelastic properties of the beam material are taken into account. The problem posed includes the differential equation of vibrations, initial conditions for the bent axis of the beam and for the added mass, boundary conditions. The conditions on the moving boundary are written as ratios between the values of the function and its derivatives to the left and right of the boundary.

Keywords: oscillations of a beam, boundary conditions, variational principles, moving spring support.

Among all the many problems of the dynamics of elastic systems from the point of view of technical applications, the problems of oscillations in systems with moving boundaries: longitudinal-transverse vibrations of the ropes of hoisting installations [1–6] flexible transmission links [2], rods of solid fuel and beams of variable length, drill strings, railway contact network [7], belt conveyors, etc.

In a mathematical setting, this is reduced to new problems in mathematical physics – to the study of the corresponding equations of hyperbolic type in variable ranges of variation of both arguments [4–6, 8–10].

Until now, there is no general approach to the formulation of such problems, and the authors in each specific case adapt the existing methods to solve the problem under consideration. Here we note that the methods for solving these equations in variable geometric domains are qualitatively different from the classical methods of mathematical physics. In other words, the studied dynamic process develops over time.

The problems of oscillation of systems with moving boundaries have been solved mainly with a linear setting and rigid fixation of boundaries, when there is no energy exchange across the boundary [1–3, 7, 11]. In rare cases, the effect of damping forces was taken into account. Real technical objects are much more complicated. Problems about vibrations of a beam with a moving support belong to a wide class of problems related to the vibrations of objects with moving boundaries. In all the cases considered earlier, the rigid fastening of the moving support excluded the exchange of energy through it. In the presence of energy exchange, the complexity in recording the conditions at the moving boundary increases. In this paper, to formulate the problem, it is proposed to use the variational principle of Hamilton.

In connection with the intensive development of numerical methods, it became possible to de-scribe such objects more accurately, taking into account a large number of factors.

Of all the possible laws of motion, in fact, one is realized for which the action

$$\int_{t_0}^{t_1} (T - U)dt \qquad (1)$$

where T and U are the kinetic and potential energies of the system takes a stationary value [10].

A new nonlinear mathematical model of transverse vibrations of a viscoelastic beam with a moving spring–loaded support carrying an attached mass is formulated. The boundary conditions are obtained in the case of the interaction of the parts of the object to the left and to the right of the moving boundary, taking into account the energy exchange through it. The resulting mathematical model makes it possible to describe high–intensity oscillations of systems with moving boundaries. Note that there are currently no methods for the analytical solution of the problem posed, so this problem, apparently, can be solved only by numerical methods.

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